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Inflation and Monopoles in Supersymmetric $\mathbf{SU(4)_c \times SU(2)_L \times SU(2)_R}$

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Abstract

We show how hybrid inflation can be successfully realized in a supersymmetric model with gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. By including a non-renormalizable superpotential term, we generate an inflationary valley along which G_{PS} is broken to the standard model gauge group. Thus, catastrophic production of the doubly charged magnetic monopoles, which are predicted by the model, cannot occur at the end of inflation. The results of the cosmic background explorer can be reproduced with natural values (of order 10^{-3}) of the relevant coupling constant, and symmetry breaking scale of G_{PS} close to 10^{16} GeV. The spectral index of density perturbations lies between unity and 0.94. Moreover, the μ-term is generated via a Peccei-Quinn symmetry and proton is practically stable. Baryogenesis in the universe takes place via leptogenesis. The low deuterium abundance constraint on the baryon asymmetry, the gravitino limit on the reheat temperature and the requirement of almost maximal $\nu_\mu - \nu_\tau$ mixing from SuperKamiokande can be simultaneously met with m_{ν_μ}, m_{ν_τ} and heaviest Dirac neutrino mass determined from the large angle MSW resolution of the solar neutrino problem, the SuperKamiokande results and $SU(4)_c$ symmetry respectively.

1 Introduction

After the recent discovery of neutrino oscillations by the SuperKamiokande experiment [1], supersymmetric (SUSY) models with left-right symmetric gauge groups have attracted a great deal of attention. These models provide a natural framework for implementing the seesaw mechanism [2] which explains the existence of the small neutrino masses. The implications of these models have been considered in Ref.[3], in the case of the gauge group $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and in Ref.[4] for the SUSY Pati-Salam (PS) [5] model based on the gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. It was shown that they lead to a constraint version of the minimal supersymmetric standard model (MSSM).

Hybrid inflation [6] has been extensively studied [7, 8, 9] in the case of the SUSY model based on the gauge group G_{LR}. However, in trying to extend this scheme to G_{PS}, we encounter the following difficulty. The spontaneous symmetry breaking of G_{PS} to the standard model gauge group G_{SM} leads to the existence of topologically stable magnetic monopole solutions. This is due to the fact that the second homotopy group of the vacuum manifold $\pi_2(G_{PS}/G_{SM})$ is non-trivial and equal to the set of integers Z. These monopoles carry two units of Dirac magnetic charge [10]. Inflation is terminated abruptly when the system reaches a critical point (instability) on the inflationary trajectory and is followed by a 'waterfall' regime during which the spontaneous breaking of G_{PS} occurs. The appropriate Higgs fields develop their non-zero vacuum expectation values (vevs) starting from zero and they can end up at any point of the vacuum manifold with equal probability. As a consequence, magnetic monopoles are copiously produced [11] by the Kibble mechanism [12] leading to a cosmological disaster.

In this paper, we propose a specific SUSY model based on G_{PS} which avoids this cosmological catastrophe. This is achieved by including a non-renormalizable term to the part of the superpotential involving the inflaton system and causing the breaking of G_{PS}. It is worth mentioning that an analogous non-renormalizable term was also used in Ref.[11] for the same purpose. In that case, however, the leading renormalizable term was eliminated by imposing a discrete symmetry. Here, we keep this leading term along with the non-renormalizable contribution. The picture that emerges turns out to be considerably different. In particular, there exists a non-trivial (classically) flat direction along which G_{PS} is spontaneously broken with the appropriate Higgs fields acquiring constant values. This direction can be used as inflationary trajectory with the necessary inclination obtained from one-loop radiative corrections [13] in contrast to the model of Ref.[11], where a classical inclination was present. Another difference is that here the termination of inflation is abrupt (as in the original hybrid inflationary scenario) and not smooth as in Ref.[11]. Nevertheless, no magnetic monopoles are formed in this transition since G_{PS} is already broken during inflation.

We show that, for a certain range of parameters, the system always passes from the above mentioned inflationary trajectory before falling into the SUSY vacuum. Thus, the magnetic monopole problem is solved for all initial conditions. It is interesting to note that the idea of breaking the gauge symmetry before (or during) inflation in order to avoid monopoles was also employed in Ref.[14]. However, the monopole problem was solved only for a certain (wide) class of initial values of the fields.

The constraints on the quadrupole anisotropy of the cosmic microwave background radiation from the cosmic background explorer (COBE) [15] measurements can be easily met with natural values (of order 10^{-3}) of the relevant coupling constant and a grand unification theory (GUT) scale close to (but somewhat smaller than) the SUSY GUT scale. Note that the mass scale in the model of Ref.[13], which uses only renormalizable couplings in the inflationary superpotential, is considerably smaller. Our model possesses a number of other interesting features too. The μ-problem of MSSM is solved [16] via a Peccei-Quinn (PQ) symmetry which also solves the strong CP problem. Although the baryon (B) and lepton (L) numbers are explicitly violated, the proton life time is considerably higher than the present experimental limits. Light neutrinos acquire masses by the seesaw mechanism and the baryon asymmetry of the universe can be generated through a primordial leptogenesis [17]. The gravitino constraint [18] on the reheat temperature, the low deuterium abundance limits [19] on the baryon asymmetry of the universe and the requirement of almost maximal $\nu_\mu - \nu_\tau$ mixing from SuperKamiokande [1] can be met for μ- and τ-neutrino masses restricted by SuperKamiokande and the large angle MSW solution of the solar neutrino puzzle respectively. The required values of the relevant coupling constants are more or less natural.

The plan of the paper is as follows. In Sec.2, we introduce our SUSY model which is based on the gauge group G_{PS} and motivate the inclusion of a non-renormalizable coupling in the inflaton sector of the theory. The full superpotential and its global symmetries are then discussed together with the solution of the μ-problem via the PQ symmetry of the model. In Sec.3, the hybrid inflationary scenario is studied in detail in this model. The structure of the potential is carefully analyzed. We calculate the one-loop radiative corrections along the inflationary trajectory by first deriving (see Appendix) the mass spectrum of the theory during inflation. The parameters of the model are then restricted by employing COBE results. In Sec.4, we discuss the reheating process following inflation, neutrino masses and mixing and baryogenesis via leptogenesis. We show that all the relevant constraints can be satisfied with natural values of the coupling constants. We summarize our conclusions in Sec.5.

2 A supersymmetric $\mathbf{SU(4)_c \times SU(2)_L \times SU(2)_R}$ model

In the SUSY PS model, the left-handed quark and lepton superfields are accommodated in the following representations:

$$\begin{aligned} F_i &= (4,2,1) \equiv \begin{pmatrix} u_i & u_i & u_i & \nu_i \\ d_i & d_i & d_i & e_i \end{pmatrix}, \\ F_i^c &= (\bar{4},1,2) \equiv \begin{pmatrix} u_i^c & u_i^c & u_i^c & \nu_i^c \\ d_i^c & d_i^c & d_i^c & e_i^c \end{pmatrix}, \end{aligned} \tag{1}$$

where the subscript $i=1,2,3$ denotes the family index [5]. The G_{PS} gauge symmetry can be spontaneously broken to G_{SM} by a pair of Higgs superfields

$$\begin{aligned} H^c &= (\bar{4},1,2) \equiv \begin{pmatrix} u_H^c & u_H^c & u_H^c & \nu_H^c \\ d_H^c & d_H^c & d_H^c & e_H^c \end{pmatrix}, \\ \bar{H}^c &= (4,1,2) \equiv \begin{pmatrix} \bar{u}_H^c & \bar{u}_H^c & \bar{u}_H^c & \bar{\nu}_H^c \\ \bar{d}_H^c & \bar{d}_H^c & \bar{d}_H^c & \bar{e}_H^c \end{pmatrix} \end{aligned} \tag{2}$$

acquiring non-vanishing vevs in the right-handed neutrino direction, $|\langle \nu_H^c \rangle|$, $|\langle \bar{\nu}_H^c \rangle| \neq 0$.

The two low energy Higgs doublets of the MSSM are contained in the following representation:

$$h = (1,2,2) \equiv \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix}. \tag{3}$$

After G_{PS} breaking, the bidoublet Higgs field h splits into two Higgs doublets h_1, h_2, whose neutral components subsequently develop weak vevs $\langle h_1^0 \rangle = v_1$ and $\langle h_2^0 \rangle = v_2$ with $\tan\beta = v_2/v_1$.

The breaking of G_{PS} can be achieved by introducing a gauge singlet superfield S, which has a trilinear (renormalizable) coupling to H^c, $\bar{H}^c$. The resulting scalar potential automatically possesses an in-built (classically) flat direction along which inflation can take place [20] with the system driven by an inclination from one-loop radiative corrections [13]. The G_{PS} gauge symmetry is restored along this trajectory and breaks spontaneously only at the end of inflation when the system falls towards the SUSY vacua. This transition leads to a cosmologically unacceptable copious production of doubly charged magnetic monopoles [11]. One way to resolve this problem is to use, for inflation, another flat direction in which G_{PS} is already broken. Such a trajectory naturally appears if we include the next order non-renormalizable superpotential coupling of S to H^c, $\bar{H}^c$ too. We find that, together with the usual flat direction with unbroken G_{PS}, an extra flat trajectory along which G_{PS} is spontaneously broken to G_{SM} emerges. The termination of inflation can then take place with G_{PS} already broken and no monopoles being produced.

An important issue is the generation of the μ-term of MSSM. This could be achieved [21] by coupling S to the electroweak Higgs superfields and using the fact that S, after gravity-mediated SUSY breaking, develops a vev. However, this is not totally satisfactory since the inflaton decays into electroweak Higgs superfields via an unsuppressed (renormalizable) coupling. As a consequence, the gravitino constraint [18] on the reheat temperature implies [22] unnaturally small values for the relevant coupling constants (of order 10^{-6} or so). We thus prefer to follow here Ref.[16] and impose a PQ symmetry on the superpotential by introducing a pair of gauge singlet superfields N, $\bar{N}$. The PQ breaking occurs at an intermediate scale by the vevs of N, $\bar{N}$ and the μ-term is generated via a non-renormalizable coupling of N and H^c, $\bar{H}^c$. The inflaton can be made to decay into right-handed neutrinos by introducing into the scheme a non-renormalizable superpotential coupling of $\bar{H}^c$ to F^c_i and the gravitino constraint can be satisfied with more natural values of the parameters. Finally, in order to give superheavy masses to d^c_H and $\bar{d}^c_H$, we introduce [23] an $SU(4)_c$ 6-plet superfield $G = (6, 1, 1)$ which, under G_{SM}, splits into $g^c = (\bar{3}, 1, 1/3)$ and $\bar{g}^c = (3, 1, -1/3)$.

The superpotential of the model, which incorporates all the above couplings, is

$$\begin{aligned} W &= \kappa S(\bar{H}^c H^c - M^2) - \beta S \frac{(\bar{H}^c H^c)^2}{M_S^2} \\ &+ \lambda_1 \frac{N^2 h^2}{M_S} + \lambda_2 \frac{N^2 \bar{N}^2}{M_S} + \lambda_{ij} F^c_i F_j h \end{aligned}$$

$$+ \quad \gamma_i \frac{\bar{H}^c \bar{H}^c}{M_S} F_i^c F_i^c + aGH^cH^c + bG\bar{H}^c\bar{H}^c, \tag{4}$$

where $M_S \sim 5 \times 10^{17}$ GeV is a superheavy string scale. Also, M, κ, $\lambda_{1,2}$, γ_i, a and b can be made positive by field redefinitions, while β is chosen positive for simplicity (it could be genuinely complex). Here, we are in the basis where γ's are diagonal.

In addition to G_{PS}, the superpotential in Eq.(4) possesses two global anomalous symmetries, a R symmetry $U(1)_R$ and a PQ symmetry $U(1)_{PQ}$. The R and PQ charges of the superfields are assigned as follows:

$$\begin{aligned} R: \quad & H^c(0), \bar{H}^c(0), S(1), G(1), F(1/2), F^c(1/2), N(1/2), \bar{N}(0), h(0); \\ PQ: \quad & H^c(0), \bar{H}^c(0), S(0), G(0), F(-1), F^c(0), N(-1), \bar{N}(1), h(1). \end{aligned} \tag{5}$$

To avoid undesirable mixing of F and h or F^c and H^c, we also impose a discrete Z_2^{mp} symmetry (know as 'matter parity'), under which F and F^c change sign.

Additional superpotential terms allowed by the symmetries of the model are

$$FFH^cH^c\bar{N}^2, \quad FFH^cH^chh, \quad FF\bar{H}^c\bar{H}^c\bar{N}^2, \quad FF\bar{H}^c\bar{H}^chh, \quad F^cF^cH^cH^c, \tag{6}$$

modulo arbitrary multiplications by non-negative powers of the combination $H^c\bar{H}^c$ (this applies to the terms in Eq.(4) too). Note that the $SU(4)_c$ indices in all couplings except the last three in Eq.(6) are contracted between 4's and $\bar{4}$'s, while in these three terms the four $SU(4)_c$ indices of the 4's or $\bar{4}$'s are contracted with an ϵ_{ijkl}. The soft SUSY breaking and instanton effects explicitly break $U(1)_R$ to Z_2, under which $N \to -N$, and $U(1)_{PQ}$ to Z_6. These two discrete symmetries are spontaneously broken by the vevs of $N, \bar{N}$ and would create a domain wall problem if the PQ transition took place after inflation. When H^c, $\bar{H}^c$, N and $\bar{N}$ acquire non-vanishing vevs, the symmetry which is left unbroken is $G_{SM} \times Z_2^{mp}$.

We can assign baryon number $1/3(-1/3)$ to all color triplets (antitriplets). Recall that there are (anti)triplets not only in F, F^c but also in $H^c, \bar{H}^c, G$. Lepton number is then defined via $B - L$. B (and L) violation comes from the last three terms in Eq.(6) which give couplings like $u^c\ d^c\ d^c_H\ \nu^c_H$ (or $u^c\ d^c\ u^c_H\ e^c_H$), $u\ d\ \bar{d}^c_H\ \bar{\nu}^c_H$ (or $u\ d\ \bar{u}^c_H\ \bar{e}^c_H$) with appropriate coefficients. Also, the terms GH^cH^c and $G\bar{H}^c\bar{H}^c$ in the superpotential give rise to the B (and L) violating couplings $g^c\ u^c_H\ d^c_H$, $\bar{g}^c\ \bar{u}^c_H\ \bar{d}^c_H$. All other combinations are B (and L) conserving since 4's are contracted with $\bar{4}$'s.

The dominant contribution to proton decay comes from effective dimension five operators generated by one-loop diagrams with two of the u^c_H, d^c_H or one of the u^c_H, d^c_H and one of the ν^c_H, e^c_H circulating in the loop. The amplitudes corresponding to these operators are estimated to be at most of order $m_{3/2}M_{GUT}/M_S^3 \lesssim 10^{-34}$ GeV^{-1} ($m_{3/2}$ is the gravitino mass). This makes the proton practically stable. Furthermore, the dominant contribution to the Majorana mass term of light neutrinos comes from FFH^cH^chh and is utterly small. So the seesaw mechanism is the only source of light neutrino masses.

The μ-term is generated, as mentioned, by a non-renormalizable superpotential coupling which contains the electroweak Higgs and the N superfield after the breaking of $U(1)_{PQ}$ by $\langle N\rangle$, $\langle \bar{N}\rangle$. The relevant part of the scalar potential for the PQ breaking is given by [16]

$$V_{PQ} = 2|N|^2 m_{3/2}^2 \left(4\lambda_2^2 \frac{|N|^4}{m_{3/2}^2 M_S^2} - |A|\lambda_2 \frac{|N|^2}{m_{3/2}M_S} + 1\right), \tag{7}$$

where A is the dimensionless coefficient of the corresponding trilinear soft SUSY breaking term. Here, the phases ϵ, θ and $\bar{\theta}$ of A, N and $\bar{N}$ are taken to satisfy the relation $\epsilon + 2\theta + 2\bar{\theta} = \pi$ and $|N|$, $|\bar{N}|$ are assumed equal which minimizes the potential. For $|A| > 4$, the absolute minimum of this potential is given by [16]

$$|\langle N\rangle| = |\langle \bar{N}\rangle| = (m_{3/2}M_S)^{1/2} \left(\frac{|A| + \sqrt{|A|^2 - 12}}{12\lambda_2}\right)^{1/2}. \tag{8}$$

Hence the PQ symmetry breaking scale is of order $\sqrt{m_{3/2}M_S} \simeq 10^{10} - 10^{11}$ GeV and the μ-term of the MSSM is $\sim m_{3/2}$ as desired.

Note that the zero temperature PQ potential (in Eq.(7)), shown in Fig.1, possesses two local minima, the trivial one at $|N| = 0$ and the PQ minimum which, for $|A| > 4$, is the absolute minimum . These minima are separated by a sizable potential barrier which prevents a successful transition from the trivial to the PQ vacuum. Taking the one-loop temperature corrections [24] to the potential into account, one can show that the PQ vacuum remains the absolute minimum at least for temperatures below the reheating temperature $T_r \sim 10^9$ GeV. The trivial vacuum is still protected by a potential barrier. We, thus, conclude that if, after inflation, the system emerges in the trivial vacuum the completion of the PQ transition will be practically impossible. We are obliged to assume that the PQ symmetry is already broken before or during inflation. The PQ vacuum then remains stable after inflation and reheating. There is yet another reason which disfavors a PQ transition after inflation. The vevs of N, $\bar{N}$ break spontaneously the Z_2 symmetry ($N \to -N$) and the Z_6 subgroup of $U(1)_{PQ}$ which is left unbroken by instantons. This would lead to disastrous domain walls in the universe.

3 The inflationary scenario

The part of the superpotential in Eq.(4) which is relevant for inflation is given by

$$\delta W = \kappa S(\bar{H}^c H^c - M^2) - \beta \frac{S(\bar{H}^c H^c)^2}{M_S^2}, \tag{9}$$

where M is a superheavy mass scale close to the GUT scale. Note that the rest of the superpotential in Eq.(4) does not affect the inflationary dynamic. The scalar potential obtained from δW is given by

$$\begin{aligned} V &= \left| \kappa(\bar{H}^c H^c - M^2) - \beta \frac{(\bar{H}^c H^c)^2}{M_S^2} \right|^2 + \kappa^2 |S|^2 (|H^c|^2 + |\bar{H}^c|^2) \left| 1 - \frac{2\beta}{\kappa M_S^2} \bar{H}^c H^c \right|^2 \\ &+ \mathrm{D-terms}, \end{aligned} \tag{10}$$

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. Vanishing of the D-terms is achieved with $|\bar{H}^c| = |H^c|$ ($\bar{H}^c$ (H^c) lies in the $\bar{\nu}_H^c$ (ν_H^c) direction). Restricting ourselves to this direction and performing an appropriate R transformation, we can bring the complex field S to the real axis, $S = \sigma/\sqrt{2}$, where σ is a normalized real scalar field. An $|S|$-independent flat trajectory suitable for hybrid inflation driven by radiative corrections is obtained in the direction $\arg(H^c) + \arg(\bar{H}^c) = 0$ along which the potential takes the form

$$V = \left[\kappa(|H^c|^2 - M^2) - \beta \frac{|H^c|^4}{M_S^2} \right]^2 + \kappa^2 \sigma^2 |H^c|^2 \left[1 - \frac{2\beta}{\kappa M_S^2} |H^c|^2 \right]^2. \tag{11}$$

We can now rewrite this potential in terms of the dimensionless variables

$$w = \frac{|S|}{M}, \qquad y = \frac{|H^c|}{M}.$$

We get

$$\tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2 y^2 (1 - 2\xi y^2)^2, \tag{12}$$

where $\xi = \beta M^2/\kappa M_S^2$. For constant w^2, this potential has the following extrema

$$
\begin{aligned}
y_1 &= 0, && (13)\\
y_2 &= \sqrt{\frac{1}{2\xi}}, && (14)\\
y_{3\pm}^2 &= \frac{1}{2\xi}\left[(1-6\xi w^2) \pm \sqrt{(1-6\xi w^2)^2 - 4\xi(1-w^2)}\right]. && (15)
\end{aligned}
$$

Note that the first two extrema are σ-independent. As it turns out, y_1 is a local minimum (maximum) if $w > (<)1$, while y_2 is a local minimum (maximum) if $w^2 > (<)\ \rho_0 = 1/8\xi - 1/2$. Inflation will take place when the system is trapped along the y_2 minimum. We restrict ourselves to $\xi < 1/4$ since, in this case, the inflationary trajectory (at y_2) is destabilized in the direction of the real part of $H^c\bar{H}^c$ before w reaches zero. Inflation can then be terminated with the system falling towards the SUSY vacua (see below) following the direction $\arg(H^c) + \arg(\bar{H}^c) = 0$, where the potential is given by Eq.(12). The potential at y_2 is $\tilde{V}_2 = (1/4\xi - 1)^2$, while at y_1 is $\tilde{V}_1 = 1$. So that, for $\xi > (<)1/8$, the extremum at y_1 lies higher (lower) than the one at y_2.

For $1/4 > \xi > 1/7.2$, the discriminant Δ appearing under the square root in Eq.(15) is negative if w^2 lies between the positive numbers $\rho_\pm = (2 \pm \sqrt{36\xi - 5})/18\xi$ and non-negative otherwise. So, for $\rho_- < w^2 < \rho_+$, the extrema at $y_{3\pm}$ do not exist. Note, however, that $\Delta \geq 0$ does not necessarily guarantee the presence of these extrema. This requires that, in addition, the right hand side of Eq.(15) is non-negative. An important ingredient on which this requirement depends is the sign of the expression $1 - 6\xi w^2$ which is positive (negative) for $w^2 < (>)\rho_1 = 1/6\xi$. One can show that, for $1/4 > \xi > 1/7.2$, $0 < \rho_0 < \rho_- < \rho_+ \leq 1$ (equality holds for $\xi = 1/6$) and $\rho_0 < \rho_1$. Also, $\rho_+ > (<)\rho_1$ for $\xi > (<)1/6$.

For $\xi \leq 1/7.2$, we always have $\Delta \geq 0$. In addition, note that $\rho_0 < (>)1$ for $\xi > (<)1/12$, $\rho_1 < (>)1$ for $\xi > (<)1/6$ and $0 < \rho_0 < \rho_1$ always. It is interesting to point out that, at $w^2 = \rho_0$, above which the extremum at y_2 turns into a local minimum, y_{3+} coincides with y_2. The minima at $y_{3\pm}$, for $w^2 = 0$, become supersymmetric, i.e., $\tilde{V}(w^2 = 0, y = y_{3\pm}) = 0$. For reasons to become obvious later, we consider the y_{3-} minimum at $w^2 = 0$ as the relevant SUSY vacuum of the theory.

Taking into account these facts, we can distinguish five cases with qualitatively different structure of the potential:

(i) For $1/4 > \xi > 1/6$, we have $0 < \rho_0 < \rho_- < \rho_1 < \rho_+ < 1$. One can then show that, for fixed $w^2 > 1$, there exist two local minima at y_1 and y_2 (the interesting inflationary trajectory) and a local maximum at y_{3+} between them (see Fig.2). For w^2 between 1 and ρ_-, the trivial extremum at y_1 becomes a local maximum and the extremum at y_{3+} disappears (see Fig.3). In this range of w^2, the system can freely fall into the desirable (inflationary) minimum at y_2 even if it was initially along the trivial trajectory at y_1 (remember that the extremum at y_2 lies lower than the one at y_1 in this case). As we further decrease w^2 to become smaller than ρ_-, a pair of two new extrema, a local minimum at y_{3-} and a local maximum at y_{3+}, are created between y_1 and y_2. As w^2 crosses ρ_0, the local maximum at y_{3+} crosses y_2 becoming a local minimum (see Fig.4). At the same time, the local minimum at y_2 turns into a local maximum and inflation is terminated with the system falling into the local minimum at y_{3-} which at $w^2 = 0$ develops into a SUSY vacuum (see below).

(ii) For $1/6 > \xi > 1/7.2$, we have $0 < \rho_0 < \rho_- < \rho_+ < 1 < \rho_1$. The situation for $w^2 > 1$ and $w^2 < \rho_-$ is similar to the previous case. For w^2 between 1 and ρ_+, the y_1 extremum becomes a local maximum and a local minimum at y_{3-} appears between y_1 and y_{3+}. As w^2 decreases below ρ_+, the extrema at $y_{3\pm}$ disappear and there exists no obstacle for the system to fall to y_2 even if it was initially at y_1. The extrema at $y_{3\pm}$ reappear as w^2 becomes smaller than ρ_-.

(iii) For $1/7.2 > \xi > 1/8$, $0 < \rho_0 < 1 < \rho_1$. The behavior of the potential for $w^2 > 1$ and $w^2 < \rho_0$ is similar to the previous cases. For $1 > w^2 > \rho_0$, however, the extremum at y_1 becomes a local maximum and a local minimum at y_{3-} appears between y_1 and y_{3+}. Notice that, in this case, although the extremum at y_2 lies lower than the one at y_1, there is no range of w^2 where the system can fall into y_2 if it was initially at y_1. Instead, it ends up directly in y_{3-} from y_1 and monopoles can be copiously produced. Of course, if the system happens to be at y_2 from the beginning, there is no production of monopoles.

(iv) For $1/8 > \xi > 1/12$, the situation is exactly as in case (iii) with the only difference that the extremum at y_2 now lies higher than the one at y_1.

(v) For $1/12 > \xi$, we have $0 < 1 < \rho_0 < \rho_1$. It turns out that, for $w^2 > \rho_0$, the local minima at y_1 and y_2 (which lies higher) are again separated by a local maximum at y_{3+}. As w^2 crosses ρ_0, the y_{3+} local maximum turns into minimum and crosses y_2 which becomes a local maximum. There is then no obstacle to keep the system from falling into y_1 even if it was at y_2. Subsequently, when w^2 becomes smaller than 1, y_1 turns into a local maximum and the system falls into y_{3-} with a copious production of magnetic monopoles.

We will restrict ourselves here to the first two cases above ($1/4 > \xi > 1/7.2$). We saw that, in these cases, even if the system starts along the trivial valley at y_1, it always falls into the (classically) flat direction at y_2. The relevant part of inflation can then take place along this trajectory with the inflaton being driven by radiative corrections [13]. So, G_{PS} is already broken during inflation and there is no production of magnetic monopoles at the end of inflation where the system falls into the y_{3-} minimum. Case (*iii*) could also solve the monopole problem provided the system starts at y_2. Case (*iv*), although quite similar to case (*iii*), is more tricky requiring further study. The reason is that, since y_2 lies higher than y_1, the system oscillates over the local maximum at y_1 after falling from y_2. Finally, case (v) is always unacceptable since the system, for all initial conditions, falls to y_{3-} from y_1 and the copious production of monopoles is unavoidable.

As we already mentioned, after inflation ends, the system falls into the minimum at y_{3-} which, at $w^2 = 0$, develops into the final SUSY vacuum of the theory. However, the system could, in principle, fall into the minimum at y_{3+} which appears only after the instability of the inflationary trajectory at $w^2 = \rho_0$ is reached. (The minimum at y_{3+} also develops into a SUSY vacuum at $w^2 = 0$.) We will argue that this does not happen. For the values of the parameters used here, the potential barrier separating the inflationary path at y_2 and the minimum at y_{3-} is considerably reduced in the last e-folding or so. (The peak of this barrier coincides with the maximum at y_{3+}.) As a consequence, the rate per unit volume and time of forming bubbles of the y_{3-} minimum ceases to be exponentially suppressed. An order of magnitude estimate then shows that the decay of the false vacuum at y_2 to the minimum at y_{3-} is completed within a fraction of one e-folding. This happens before the appearance of the minimum at y_{3+}, i.e., before the system reaches the critical point at $w^2 = \rho_0$ (but very close to this point). Moreover, in the last stages of inflation, the above barrier is small enough to be overcome by the inflationary density perturbations. This can also accelerate the completion of this transition.

To avoid confusion we should mention here that ξ is not an extra free parameter. It depends on the coupling κ and the superheavy mass scale M (we put $\beta = 1$). The values of κ and M will be related by calculating the quadrupole anisotropy of the cosmic microwave background radiation $(\delta T/T)_Q$ as a function of the number of e-foldings of our present horizon N_Q and compare it with the measurements of COBE [15]. The parameter ξ then becomes a function of the basic coupling constant of the scheme κ. So, searching for solutions with ξ in the desirable range and also satisfying all the other requirements which we will discuss below is a highly non-trivial task.

As already mentioned, the interesting part of inflation takes place when the system is trapped along the trajectory at y_2. Inflation is driven by the constant classical energy density on this trajectory which also breaks SUSY. This breaking gives rise to non-trivial radiative corrections [13, 25] which lift the (classical) flatness of this trajectory producing a necessary inclination for driving the inflaton towards the SUSY vacua. As will be seen later, the slow-roll conditions [26] are satisfied and inflation continues essentially till w^2 reaches ρ_0, where the inflationary trajectory is destabilized. To calculate the one-loop radiative corrections at y_2 we need to construct the mass spectrum of the theory on this path where both G_{PS} and SUSY are broken. Details of the calculation can be found in the Appendix. We summarize the results in the Table.

Fields	Squared Masses
2 real scalars	$4\kappa^2\|S\|^2 \mp 2\kappa^2 M^2(\frac{1}{4\xi}-1)$
1 Majorana fermion	$4\kappa^2\|S\|^2$
1 real scalar	$5g^2v^2/2$
1 gauge boson	$5g^2v^2/2$
1 Dirac fermion	$5g^2v^2/2$
8 real scalars	g^2v^2
8 gauge bosons	g^2v^2
8 Dirac fermions	g^2v^2
6 complex scalars	$4a^2v^2$
3 Dirac fermions	$4a^2v^2$
6 complex scalars	$4b^2v^2$
3 Dirac fermions	$4b^2v^2$

Table: The mass spectrum of the model as the system moves along the inflationary trajectory at y_2. The parameter $v = (\kappa M_S^2/2\beta)^{1/2}$ is the vev of ν_H^c, $\bar{\nu}_H^c$ on this trajectory and g is the G_{PS} gauge coupling constant.

Using this spectrum, we can now calculate the one-loop radiative correction to the potential along the inflationary trajectory from the Coleman-Weinberg formula [27]

$$\Delta V = \frac{1}{64\pi^2}\sum_i (-)^{F_i}\ M_i^4 \ln\left(\frac{M_i^2}{\Lambda^2}\right), \tag{16}$$

where the sum extends over all helicity states i, F_i and M_i^2 are the fermion number and squared mass of the ith state and Λ is a renormalization mass scale. We find that the inflationary effective potential is given by

$$V_{inf}^{eff} = \kappa^2 m^4\left(1+\frac{\kappa^2}{16\pi^2}\left[2\ln\left(\frac{2\kappa^2\sigma^2}{\Lambda^2}\right)+(z+1)^2\ln(1+z^{-1})+(z-1)^2\ln(1-z^{-1})\right]\right), \tag{17}$$

where $m^2 = M^2(1/4\xi - 1)$ and $z = \sigma^2/m^2$. We see that the only non-zero contributions to the effective potential come from the $|S|$-dependent part of the spectrum (in the first two lines of the Table). This is a consequence of the fact that, along the inflationary trajectory, SUSY breaking, due to the presence of non-zero vacuum energy density, occurs only in the inflaton sector. In particular, there is mass splitting only in the supermultiplet which contains the complex scalar field $\theta = (\delta\nu_H^c + \delta\bar{\nu}_H^c)/\sqrt{2}$ (see Appendix).

Note that radiative corrections lift the (classical) flatness of the inflationary trajectory providing the necessary inclination for driving the inflaton field S towards zero. It is important to observe that although the effective potential in Eq.(17) does depend on the unknown scale Λ, its inclination (derivative with respect to σ) is Λ-independent. This is due to the fact that the supertrace of M^4 (M^2 being the mass squared matrix) appearing in Eq.(16) is, as one can readily deduce using the spectrum in the Table, σ-independent. This is an important property since otherwise $(\delta T/T)_Q$ and N_Q would depend on the unknown mass parameter Λ.

Inflation is terminated only very close to the critical point $\sigma = m$ ($z = 1$ or $w^2 = \rho_0$) after which the inflationary path is destabilized and the system falls into y_{3-}. This can be checked, for all relevant values of the coupling constants, by employing the slow-roll parameters ϵ and η [26]. It turns out that $\epsilon \ll 1$ for $z \geq 1$, while $|\eta|$ exceeds unity only for z's extremely close to 1.

The quadrupole anisotropy of the cosmic microwave background radiation can be calculated to be [9]:

$$\left(\frac{\delta T}{T}\right)_Q \simeq \pi\left(\frac{32N_Q}{45}\right)^{\frac{1}{2}}\left(\frac{m}{M_P}\right)^2 x_Q^{-1}y_Q^{-1}\Lambda(x_Q^2)^{-1}, \tag{18}$$

where $M_P = 1.22 \times 10^{19}$ GeV is the Planck scale,

$$\Lambda(z) = (z+1)\ln(1+z^{-1}) + (z-1)\ln(1-z^{-1}), \tag{19}$$

and

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z}\Lambda^{-1}(z), y_Q \geq 0, \tag{20}$$

with $x_Q = |\sigma_Q|/m$, σ_Q being the value of σ when our present horizon scale crossed outside the inflationary horizon. The coupling constant κ can be evaluated from [9]

$$\kappa = \frac{4\pi^{\frac{3}{2}}}{\sqrt{N_Q}}\frac{m}{M_P}y_Q. \tag{21}$$

Now, using the COBE constraint, $(\delta T/T)_Q = 6.6 \times 10^{-6}$ [15], taking $N_Q = 55$ and eliminating x_Q between Eqs.(18) and (21), we can obtain m and, consequently, ξ and M as functions of κ (we put $\beta = 1$ and $M_S = 5 \times 10^{17}$ GeV). The inflationary scale $v_{\rm infl} = \kappa^{1/2}m$ and the spectral index of density perturbations $n = 1 - 6\epsilon + 2\eta$ are then also found as functions of κ and are depicted in Figs.5 and 6 respectively.

The SUSY minimum can be obtained from y_{3-} in Eq.(15) by putting $w = 0$. The common vev $v_0 = |\langle\nu_H^c\rangle| = |\langle\bar{\nu}_H^c\rangle|$ of H^c and $\bar{H}^c$ at this minimum is then given by

$$(\frac{v_0}{M})^2 = \frac{1}{2\xi}(1 - \sqrt{1-4\xi}), \tag{22}$$

and is shown in Fig.7 as a function of κ.

4 Neutrino masses and lepton asymmetry

The inflaton consists of the two complex scalar fields S and θ, which have equal masses given by

$$m_{\rm infl}^2 = 2\kappa^2 v_0^2 (1 - \frac{2\xi v_0^2}{M^2})^2. \tag{23}$$

At the end of inflation, the two fields S and θ oscillate about the SUSY minimum and decay into a pair of right-handed sneutrinos (ν_i^c) and neutrinos ($\psi_{\nu_i^c}$) respectively. The masses of these (s)neutrinos are generated, after the breaking of G_{PS}, by the superpotential coupling $\gamma_i \bar{H}^c\bar{H}^c F_i^c F_i^c/M_S$ in Eq.(4) and turn out to be

$$M_i = 2\gamma_i \frac{v_0^2}{M_S}. \tag{24}$$

This same coupling together with the terms in Eq.(9) constitute the part of the superpotential which is relevant for the decay of the inflaton. We obtain the Lagrangian terms

$$L^S_{decay} = -\sqrt{2}\gamma_i \frac{v_0}{M_S}\ S^*\ \nu^c_i\ \nu^c_i\ m_{\rm infl}\ + h.c. \tag{25}$$

for the decay of S and

$$L^\theta_{decay} = -\sqrt{2}\gamma_i \frac{v_0}{M_S}\ \theta\ \psi_{\nu^c_i}\ \psi_{\nu^c_i}\ + h.c. \tag{26}$$

for the decay of θ respectively. From Eqs.(25) and (26), we deduce that S and θ have equal decay widths given by

$$\Gamma_{S\to\ \nu^c_i\nu^c_i} = \Gamma_{\theta\to\ \psi_{\nu^c_i}\psi_{\nu^c_i}} = \frac{1}{8\pi}\left(\frac{M_i}{v_0}\right)^2 m_{\rm infl}, \tag{27}$$

provided that $M_i < m_{\rm infl}/2$. To minimize the number of small couplings we take $\gamma_3 = 1$. We assume that

$$M_2 < m_{\rm infl}/2 \leq M_3 = 2v_0^2/M_S, \tag{28}$$

so that the inflaton decays into the second heaviest right-handed neutrino superfield with mass M_2. (The second inequality in this equation holds, in any case, for all relevant values of the parameters.) Thus the reheating temperature T_r after inflation, for the MSSM spectrum, is given by [7]

$$T_r = \frac{1}{7}(\Gamma M_P)^{1/2} = \frac{1}{7}\left(\frac{M_P\ m_{\rm infl}}{8\pi}\right)^{1/2}\frac{M_2}{v_0}. \tag{29}$$

The gravitino constraint [18] gives an upper bound on T_r of about 10^9 GeV for gravity-mediated SUSY breaking with universal boundary conditions. To maximize the naturalness of the model, we take the maximal value of M_2 (and thus γ_2) allowed by the gravitino constraint. Note that these values of M_2 turn out to be about two orders of magnitude lower than the corresponding values of $m_{\rm infl}/2$ and, thus, the first inequality in Eq.(28) is well satisfied.

Analysis [28] of the CHOOZ experiment [29] shows that the solar and atmospheric neutrino oscillations decouple, allowing us to concentrate on the two heaviest families. The light neutrino mass matrix is then given by

$$m_\nu = -\tilde{M}^D \frac{1}{M^R} M^D, \tag{30}$$

where M^D is the Dirac neutrino mass matrix with positive eigenvalues $m^D_{2,3}$ ($m^D_2 \leq m^D_3$), and M^R the Majorana mass matrix of the right-handed neutrinos with positive eigenvalues $M_{2,3}$ ($M_2 \leq M_3$) given in Eq.(24). The two positive eigenvalues of m_ν will be denoted by m_2 (or m_{ν_μ}) and m_3 (or m_{ν_τ}), with $m_2 \leq m_3$. The determinant and trace invariance of $m^\dagger_\nu m_\nu$ provide us with two constraints [30] on the mass parameters $m_{2,3}$, $m^D_{2,3}$, $M_{2,3}$ and the angle θ and phase δ of the rotation matrix which diagonalizes the right-handed neutrino mass matrix M^R in the basis where M^D is diagonal.

The bounds on m_{ν_μ} from the small or large angle MSW solution of the solar neutrino puzzle are respectively 2×10^{-3} eV $\leq m_{\nu_\mu} \leq 3.2 \times 10^{-3}$ eV or 3.6×10^{-3}eV $\leq m_{\nu_\mu} \leq 1.3 \times 10^{-2}$ eV [31]. As we will see below, the latter solution is favored in our model. The τ-neutrino mass is restricted in the range 3×10^{-2} eV $\leq m_{\nu_\tau} \leq 11 \times 10^{-2}$ eV from the results of SuperKamiokande [1] which also imply almost maximal $\nu_\mu - \nu_\tau$ mixing, i.e., $\sin^2 2\theta_{\mu\tau} > 0.8$. Assuming that the Dirac mixing angle θ^D (i.e., the mixing angle in the absence of right-handed neutrino Majorana masses) is negligible, we find [30] $\theta_{\mu\tau} \simeq \varphi$, where φ is the rotation angle which diagonalizes m_ν.

An important constraint comes from the baryon asymmetry of the universe. In this model, a primordial lepton asymmetry is generated [17] by the decay of the superfield ν^c_2 which emerges as the decay product of the inflaton. (This lepton asymmetry is subsequently partially converted into baryon asymmetry by electroweak sphalerons.) The superfield ν^c_2 decays into electroweak Higgs and (anti)lepton superfields. The resulting lepton asymmetry is [30]

$$\frac{n_L}{s} \simeq 1.33 \; \frac{9T_r}{16\pi m_{\rm infl}} \frac{M_2}{M_3} \; \frac{c^2 s^2 \sin 2\delta ({m^D_3}^2 - {m^D_2}^2)^2}{v_2^2({m^D_3}^2 s^2 + {m^D_2}^2 c^2)}, \tag{31}$$

where $s = \sin\theta$ and $c = \cos\theta$. This is related to the baryon asymmetry n_B/s by $n_L/s = -(79/28)(n_B/s)$ for the spectrum of the MSSM [32]. Thus, the low deuterium abundance constraint [19] on the baryon asymmetry of the universe $0.017 \leq \Omega_B h^2 \leq 0.021$ gives $1.8 \times 10^{-10} \leq -n_L/s \leq 2.3 \times 10^{-10}$.

Due to the presence of $SU(4)_c$ in G_{PS}, the Dirac mass parameter m^D_3 coincides with the asymptotic value of the top quark mass. Taking renormalization effects into account, in the context of the MSSM with large $\tan\beta$, we find [30]

$$m^D_3 = 110 - 120 \text{ GeV}. \tag{32}$$

For each value of κ, the Majorana masses $M_{2,3}$ are fixed. Taking $m_{2,3}$ and m_3^D also fixed in their allowed ranges, we are left with only three undetermined parameters δ, θ and m_2^D which are further restricted by four constraints: almost maximal $\nu_\mu - \nu_\tau$ mixing ($\sin^2 2\theta_{\mu\tau} > 0.8$), the leptogenesis restriction ($1.8\times10^{-10} \leq -n_L/s \leq 2.3\times10^{-10}$) and the constraints from the trace and determinant invariance of $m_\nu^\dagger m_\nu$. It is highly non-trivial that solutions satisfying all the above requirements can be found with natural values of κ (of order 10^{-3}) and m_2^D of order 1 GeV. Typical solution can be constructed, for instance, for $\kappa = 1.6 \times 10^{-3}$, which corresponds to $\xi \simeq 0.21$, $v_0 \simeq 1.1 \times 10^{16}$ GeV, $m_{\rm infl} \simeq 9.9 \times 10^{12}$ GeV, $M_2 \simeq 3.5 \times 10^{10}$ GeV and $M_3 \simeq 4.8 \times 10^{14}$ GeV (remember $\beta = 1$, $M_S = 5 \times 10^{17}$ GeV and $T_r = 10^9$ GeV). Taking, for example, $m_{\nu_\mu} = 1.3 \times 10^{-2}$ eV, $m_{\nu_\tau} = 6.6 \times 10^{-2}$ eV and $m_3^D = 115$ GeV, we find $m_2^D \simeq 1.5$ GeV, $\sin^2 2\theta_{\mu\tau} \simeq 0.85$, $n_L/s \simeq -2.2 \times 10^{-10}$ and $\theta \simeq 0.011$ for $\delta \simeq -\pi/16$.

It is interesting to note that the mass scale v_0 is about 10^{16} GeV which is consistent with the unification of the gauge couplings of the MSSM. Also, the values of the μ-neutrino mass, for which solutions are found, turn out to be consistent with the large rather than the small angle MSW mechanism.

5 Conclusions

We have constructed a SUSY GUT model based on the G_{PS} gauge symmetry group. This model is consistent with all particle physics and cosmological requirements. The μ-problem is solved by introducing a global anomalous PQ symmetry $U(1)_{PQ}$, which also solves the strong CP problem. Although baryon and lepton numbers are violated in the superpotential, the proton turns out to be practically stable. SUSY hybrid inflation is 'naturally' and successfully incorporated in this model but in an unconventional way. In the standard realizations of SUSY hybrid inflation, the superpotential involves only renormalizable couplings of the GUT Higgs superfields and a gauge singlet. We have modified this picture by including the next order non-renormalizable coupling too. In contrast to the usual case, inflation now takes place along a classically flat direction where the gauge symmetry (G_{PS}) is spontaneously broken to G_{SM}. As a consequence, after inflation ends, there is absolutely no production of doubly charged magnetic monopoles, which are associated with the breaking of G_{PS}. Thus, the cosmological catastrophe one would encounter by employing the usual inflationary scheme in the SUSY PS theory is avoided. Our mechanism is crucial for the viability of any model containing cosmologically disastrous topological defects such as magnetic monopoles or domain walls and leads to complete absence of such objects. It is interesting to point out that, although the usual trajectory with unbroken G_{PS} also exists, there is a range of parameters for which the system finally inflates along the non-trivial path before falling into the SUSY vacua. Thus, the monopole problem can be solved for all possible initial conditions.

The classical flatness of the inflationary valley is lifted by one-loop radiative corrections which produce an inclination for driving the inflaton towards the SUSY vacua. The measurements of COBE can be easily reproduced with natural values (of order 10^{-3}) of the relevant coupling constant. The GUT mass scale comes out a little smaller than the SUSY GUT scale but certainly much closer to it than in the standard SUSY inflationary scheme. The spectral index of density perturbations ranges between about 1 and 0.94.

After inflation ends, the inflaton oscillates about the SUSY vacuum and decays into the second heaviest right-handed neutrino superfield thereby reheating the universe. The subsequent decay of these right-handed neutrinos to lepton and electroweak Higgs superfields generates a lepton asymmetry which is then partially converted to baryon asymmetry by the electroweak instantons. We require that the so obtained baryon asymmetry of the universe is consistent with the low deuterium abundance constraint. We also take almost maximal $\nu_\mu - \nu_\tau$ mixing as indicated by SuperKamiokande. The μ- and τ-neutrino masses are restricted by the MSW resolution of the solar neutrino puzzle and the heaviest Dirac neutrino mass by $SU(4)_c$ symmetry. We find that all these requirements can be met with natural values (of order 10^{-3}) of the relevant coupling constant. Note that the second heaviest Dirac neutrino mass turns out to be of order 1 GeV and masses of ν_μ consistent with the large rather than the small angle MSW mechanism are favored.

Finally, we would like to point out that this new SUSY hybrid inflationary scenario could be extended to higher gauge groups such as $SO(10)$. The breaking of $SO(10)$ can be achieved by including, among other representations, a pair of 16, $\bar{16}$ Higgs superfields acquiring vevs in the right-handed neutrino direction. Inflation and reheating are expected to be quite similar to the ones discussed here with the only complication that more Higgs superfield representations such as 54 and 45 will be involved for gauge symmetry breaking to G_{SM}. The magnetic monopole problem can then be solved only if some of these fields are non-zero too on the inflationary trajectory.

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Appendix: Derivation of the mass spectrum during inflation

In this Appendix, we sketch the derivation of the mass spectrum of the model when the system is trapped along the inflationary trajectory at y_2. During inflation, the fields H^c, $\bar{H}^c$ acquire vevs in the ν^c_H, $\bar{\nu}^c_H$ direction which break the gauge symmetry G_{PS} down to G_{SM}. These vevs are given by $\langle\nu^c_H\rangle = \langle\bar{\nu^c_H}\rangle = v = (\kappa M_S^2/2\beta)^{1/2}$ and we can write $\nu^c_H = v + \delta\nu^c_H$ and $\bar{\nu}^c_H = v + \delta\bar{\nu}^c_H$. One can show that the scalar potential in Eq.(10) does not generate masses for the scalar components of the superfield H^c ($\bar{H}^c$) in the directions u^c_H, d^c_H, e^c_H ($\bar{u}^c_H$, $\bar{d}^c_H$, $\bar{e}^c_H$). On the contrary, a simple calculation yields that the normalized real scalar fields $\mathrm{Re}(\delta\nu^c_H + \delta\bar{\nu}^c_H)$ and $\mathrm{Im}(\delta\nu^c_H + \delta\bar{\nu}^c_H)$ acquire non-zero masses given by

$$m^2_{\pm} = 4\kappa^2|S|^2 \mp 2\kappa^2 M^2(\frac{1}{4\xi} - 1) \tag{33}$$

respectively. The superpotential in Eq.(9) gives rise to just one massive Majorana fermion with $m^2 = 4\kappa^2|S|^2$ corresponding to the direction $(\nu^c_H + \bar{\nu}^c_H)/\sqrt{2}$. We see that the SUSY breaking along the inflationary trajectory, which is due to the non-zero vacuum energy density $\kappa^2 M^4(1/4\xi - 1)^2$, produces a mass splitting in the ν^c_H, $\bar{\nu}^c_H$ supermultiplets. Actually, as we will show, this is the only place where such a mass splitting appears.

The D-term contribution to the scalar masses can be found from

$$\frac{1}{2}g^2\sum_a(\bar{H}^{c^*}T^a\bar{H}^c + H^{c^*}T^aH^c)^2, \tag{34}$$

where g is the G_{PS} gauge coupling constant and the sum extends over all the generators T^a of G_{PS}. The part of this sum over the generators $T^{15} = (1/2\sqrt{6})\ \mathrm{diag}(1,1,1,-3)$ of $SU(4)_c$ and $T^3 = (1/2)\ \mathrm{diag}(1,-1)$ of $SU(2)_R$ gives rise to a mass term for the normalized real scalar field $\mathrm{Re}(\delta\nu^c_H - \delta\bar{\nu}^c_H)$ with $m^2 = 5g^2v^2/2$ as one can show by using the above expansion of ν^c_H, $\bar{\nu}^c_H$.

The gauge bosons A^a can acquire masses from the Lagrangian terms

$$g^2(|\sum_a A^a T^a \bar{H}^c|^2 + |\sum_a A^a T^a H^c|^2). \tag{35}$$

Taking the contribution of T^{15} and T^3 again, we obtain a mass term for the normalized gauge field

$$A^\perp = -\sqrt{\frac{3}{5}}A^{15} + \sqrt{\frac{2}{5}}A^3$$

with $m^2 = 5g^2v^2/2$ (the real field $\mathrm{Im}(\delta\nu_H^c - \delta\bar{\nu}_H^c)$, which is so far left massless, is absorbed by this gauge boson).

Fermion masses get also contributions from the Lagrangian terms

$$i\sqrt{2}g\sum_a \lambda^a(\bar{H}^{c^*}T^a\psi_{\bar{H}^c} + H^{c^*}T^a\psi_{H^c}) + h.c., \tag{36}$$

where λ^a is the gaugino corresponding to T^a and $\psi_{\bar{H}^c}$, ψ_{H^c} represent the chiral fermions belonging to the superfields $\bar{H}^c$, H^c respectively. Concentrating again on T^{15}, T^3, we obtain a Dirac mass term between the chiral fermion in the superfield $(\nu_H^c - \bar{\nu}_H^c)/\sqrt{2}$ and the gaugino $-i\lambda^\perp$ with $m^2 = 5g^2v^2/2$. This completes the analysis of the ν_H^c, $\bar{\nu}_H^c$ sector together with the gauge supermultiplet in the $T^\perp$ direction.

The eight normalized real scalar fields $\mathrm{Re}(u_H^c - \bar{u}_H^{c^*})$, $\mathrm{Im}(u_H^c - \bar{u}_H^{c^*})$ (three colors), $\mathrm{Re}(e_H^c - \bar{e}_H^{c^*})$, $\mathrm{Im}(e_H^c - \bar{e}_H^{c^*})$ acquire mass terms from the D-term contribution in Eq.(34) with $m^2 = g^2v^2$. Indeed, the part of the sum in this equation over the generators

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{37}$$

of $SU(2)_R$ gives

$$\frac{1}{2}g^2v^2(\mathrm{Re}(e_H^c - \bar{e}_H^{c^*}))^2 + \frac{1}{2}g^2v^2(\mathrm{Im}(e_H^c - \bar{e}_H^{c^*}))^2. \tag{38}$$

Similarly, the sum over the $SU(4)_c$ generators T_i^1 and T_i^2 ($i = 1,2,3$) with $1/2$ ($1/2$) and $-i/2$ ($i/2$) in the $i4$ ($4i$) entries respectively and zero everywhere else generates the masses of $\mathrm{Re}(u_H^c - \bar{u}_H^{c^*})$, $\mathrm{Im}(u_H^c - \bar{u}_H^{c^*})$ (three colors). Using Eq.(35), one can show that the eight gauge bosons A^1, A^2, A_i^1, A_i^2 ($i = 1,2,3$) become massive with $m^2 = g^2v^2$ (they absorb the real fields $\mathrm{Re}(e_H^c + \bar{e}_H^{c^*})$, $\mathrm{Im}(e_H^c + \bar{e}_H^{c^*})$, and $\mathrm{Re}(u_H^c + \bar{u}_H^{c^*})$, $\mathrm{Im}(u_H^c + \bar{u}_H^{c^*})$ (three colors)).

The chiral fermions $\psi_{\bar{e}^c_H}$ and $\psi_{e^c_H}$ combine with the gauginos $\lambda^+ = (\lambda^1 + i\lambda^2)/\sqrt{2}$ and $\lambda^- = (\lambda^1 - i\lambda^2)/\sqrt{2}$ respectively to form two Dirac fermion states with $m^2 = g^2 v^2$ as one deduces from Eq.(36). Similarly, $\psi_{\bar{u}^c_H}$ and $\psi_{u^c_H}$ (three colors) together with $\lambda^+_i = (\lambda^1_i + i\lambda^2_i)/\sqrt{2}$ and $\lambda^-_i = (\lambda^1_i - i\lambda^2_i)/\sqrt{2}$ give six more Dirac fermions with the same mass squared.

The only superfields left are the d^c_H, $\bar{d}^c_H$, g^c, $\bar{g}^c$. They do not mix with the rest of the spectrum, as one can easily show, and acquire masses from the last two superpotential terms in Eq.(4). These terms can be explicitly written as

$$\begin{aligned} a\, G\, H^c\, H^c &= 2\, a\, (-d^c_H\, \nu^c_H \, + \, u^c_H\, e^c_H)\, \bar{g}^c \, + 2\, a\, u^c_H\, d^c_H\, g^c, \\ b\, G\, \bar{H}^c\, \bar{H}^c &= 2\, b\, (-\bar{d}^c_H\, \bar{\nu}^c_H \, + \, \bar{u}^c_H\, \bar{e}^c_H)\, g^c + 2\, b\, \bar{u}^c_H \bar{d}^c_H \bar{g}^c. \end{aligned} \tag{39}$$

The scalar potential then contains the terms

$$4a^2\, v^2\, (|d^c_H|^2 + |\bar{g}^c|^2) \, + \, 4\, b^2\, v^2\, (|\bar{d}^c_H|^2 + |g^c|^2) \tag{40}$$

and we obtain six complex scalars $(d^c_H, \bar{g}^c)$ with $m^2 = 4a^2 v^2$ and six complex scalars $(\bar{d}^c_H, g^c)$ with $m^2 = 4b^2 v^2$. Also, the chiral fermions $\psi_{d^c_H}$ and $\psi_{\bar{g}^c}$ combine to give three Dirac fermions with $m^2 = 4a^2 v^2$, while $\psi_{\bar{d}^c_H}$ and ψ_{g^c} give three Dirac fermions with $m^2 = 4b^2 v^2$.

We see that all the fields acquire non-zero masses but SUSY is broken only in the sector of the two real scalar fields with masses given in Eq.(33) and the Majorana fermion with $m^2 = 4\kappa^2 |S|^2$. In all other supermultiplets, the fermionic and bosonic components have equal masses. As a consequence, these supermultiplets give zero contribution to the supertraces $\mathrm{STr} M^{2n}$ for any integer $n \geq 0$ (and in general to the supertrace of any function of the mass squared matrix M^2). Thus, in calculating any such supertrace, we only have to consider the two real scalars and the Majorana fermion mentioned above. Note that, in particular, their contribution to $\mathrm{STr} M^2$ is zero.

References

[1] T. Kajita, talk given at XVIIIth International Conference on Neutrino Physics and Astrophysics (Neutrino'98), Takayama, Japan, 4-9 June, 1998.

[2] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Freeman (North Holland, Amsterdam, 1979) p. 315;
T. Yanagida, Prog. Th. Phys. B 135 (1979) 66.

[3] K. Babu, B. Dutta and R. Mohapatra, Phys. Rev. D 60 (1999) 095004;
Phys. Lett. B 458 (1999) 93; hep-ph/9905464.

[4] S. Khalil and Q. Shafi, Phys. Rev. D 61 (2000) 035003;
S. King and Q. Shafi, Phys. Lett. B 442 (1998) 135.

[5] J. C. Pati and A. Salam, Phys. Rev. D 10 (1974) 275.

[6] A. D. Linde, Phys. Rev. D 49 (1994) 748.

[7] G. Lazarides, R. Schaefer and Q. Shafi, Phys. Rev. D 56 (1997) 1324.

[8] G. Lazarides and N. D. Vlachos, Phys. Lett. B 459 (1999) 482.

[9] G. Lazarides, hep-ph/9905450 (PRHEP-trieste99/008).

[10] G. Lazarides, M. Magg and Q. Shafi, Phys. Lett. B 97 (1980) 87.

[11] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D 52 (1995) 559.

[12] T. W. B. Kibble, J. Phys. A 9 (1976) 387.

[13] G. Dvali, R. Schaefer and Q. Shafi, Phys. Rev. Lett. 73 (1994) 1886.

[14] L. Covi, G. Mangano, A. Masiero and G. Miele, Phys. Lett. B 424 (1998) 253.

[15] G. F. Smoot *et al.*, Astrophys. J. Lett. 396 (1992) L1; C. L. Bennett *et al.*, Astrophys. J. Lett. 464 (1996) 1.

[16] G. Lazarides and Q. Shafi, Phys. Rev. D 58 (1998) 071702.

[17] M. Fukugita and T. Yanagita, Phys. Lett. B 174 (1986);
W. Buchmüller and M. Plümacher, Phys. Lett. B 389 (1996) 73;
G. Lazarides and Q. Shafi, Phys. Lett. B 258 (1991) 350.
For a recent review in the context of SUSY hybrid inflation see G. Lazarides, hep-ph/9904428.

[18] M. Yu. Khlopov and A. D. Linde, Phys. Lett. B 138 (1984) 265 ;
J. Ellis, J. E. Kim and D. Nanopoulos, Phys. Lett. B 145 (1984) 181.

[19] S. Burles and D. Tytler, Astrophys. Journal 499 (1998) 699; ibid. 507 (1998) 732.

[20] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D 49 (1994) 6410.

[21] G. Dvali, G. Lazarides and Q. Shafi, Phys. Lett. B 424 (1998) 259.

[22] G. Lazarides and N. D. Vlachos, Phys. Lett. B 441 (1998) 46.

[23] I. Antoniadis and G. K. Leontaris, Phys. Lett. B 216 (1989) 333.

[24] L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320;
S. Weinberg, Phys. Rev. D 9 (1974) 3357.

[25] R. Jeannerot and J. Lesgourgues, hep-ph/9905208.

[26] A. R. Liddle and D. H. Lyth, Phys. Rep. 231 (1993) 1.

[27] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.

[28] C. Giunti, hep-ph/9802201.

[29] M. Apollonio *et al.*, Phys. Lett. B 420 (1998) 397.

[30] G. Lazarides, Q. Shafi and N. D. Vlachos, Phys. Lett. B 427 (1998) 53.

[31] J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, Phys. Rev. D 58 (1998) 096016; ibid. D 60 (1999) 093001.

[32] L. E. Ibáñez and F. Quevedo, Phys. Lett. B 283 (1992) 261.

Figures

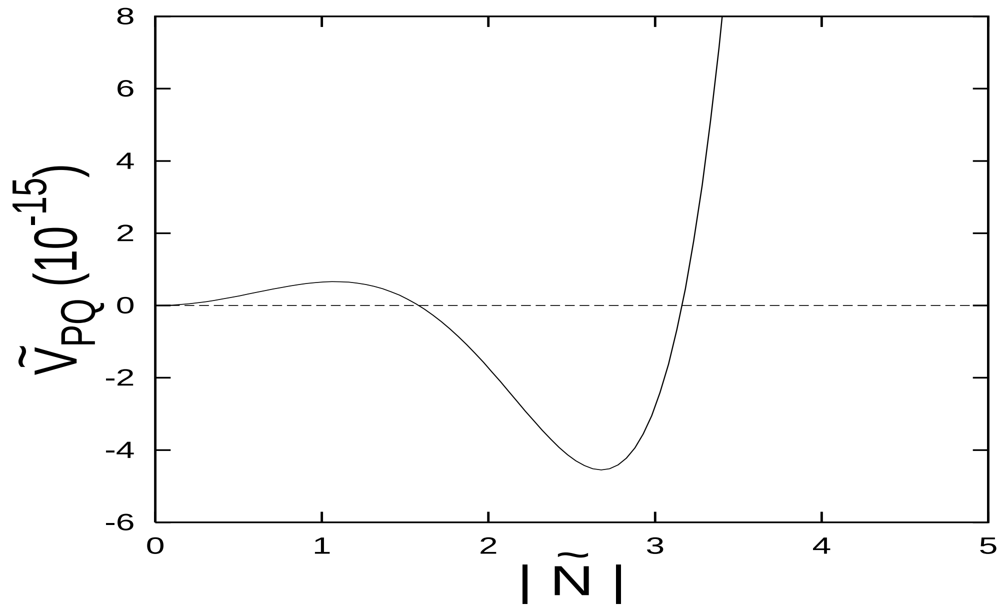

Figure 1: The (dimensionless) zero temperature potential $\tilde{V}_{PQ} = V_{PQ}/(m_{3/2}M_S)^2$ with V_{PQ} given in Eq.(7) versus $|\tilde{N}| = |N|/(m_{3/2}M_S)^{1/2}$, for $|A| = 5, \lambda_1 = 0.3, \lambda_2 = 0.1, m_{3/2} =$ 300 GeV and $M_S = 5 \times 10^{17}$ GeV ($\mu = 600$ GeV).

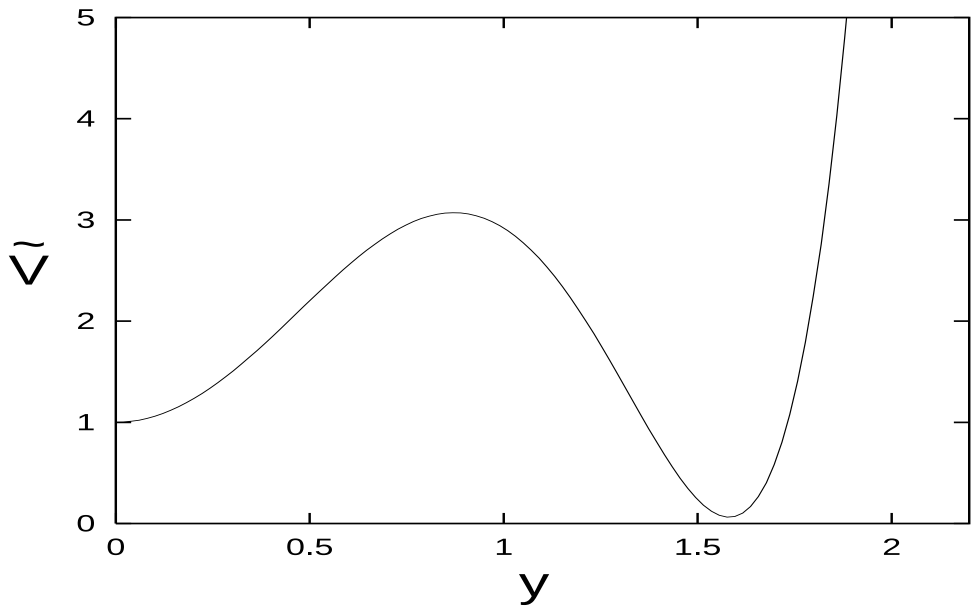

Figure 2: The (dimensionless) potential $\tilde{V}$, given in Eq.(12), versus y for $w = 2$, $\xi = 1/5$.

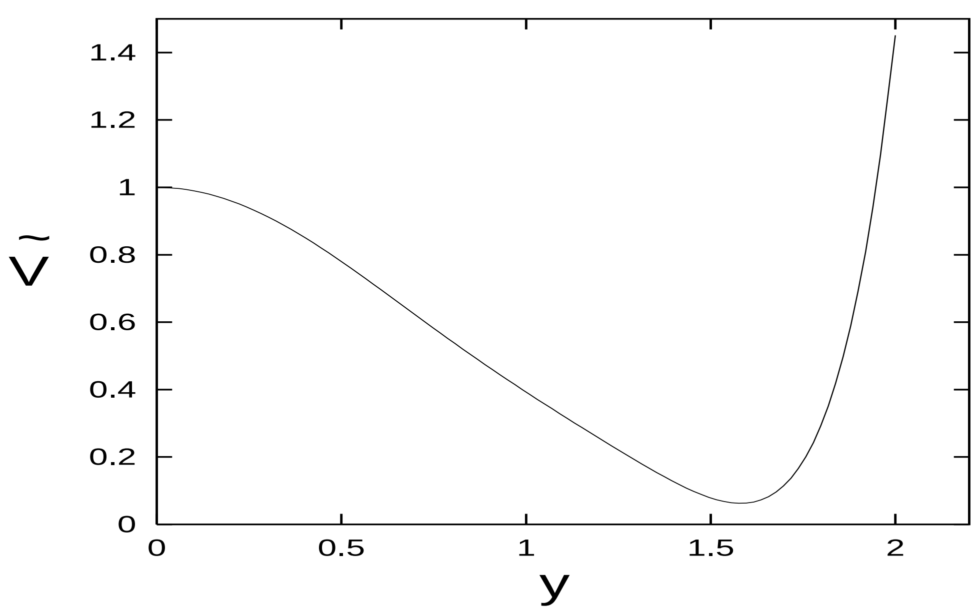

Figure 3: The (dimensionless) potential $\tilde{V}$, given in Eq.(12), versus y for $w = 0.7$, $\xi = 1/5$.

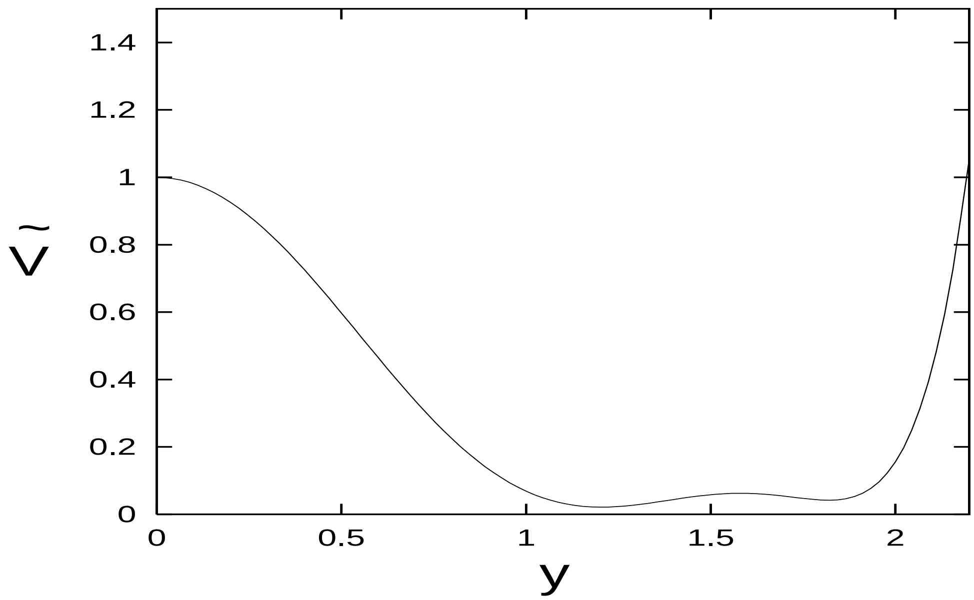

Figure 4: The (dimensionless) potential $\tilde{V}$, given in Eq.(12), versus y for $w = 0.2$, $\xi = 1/5$.

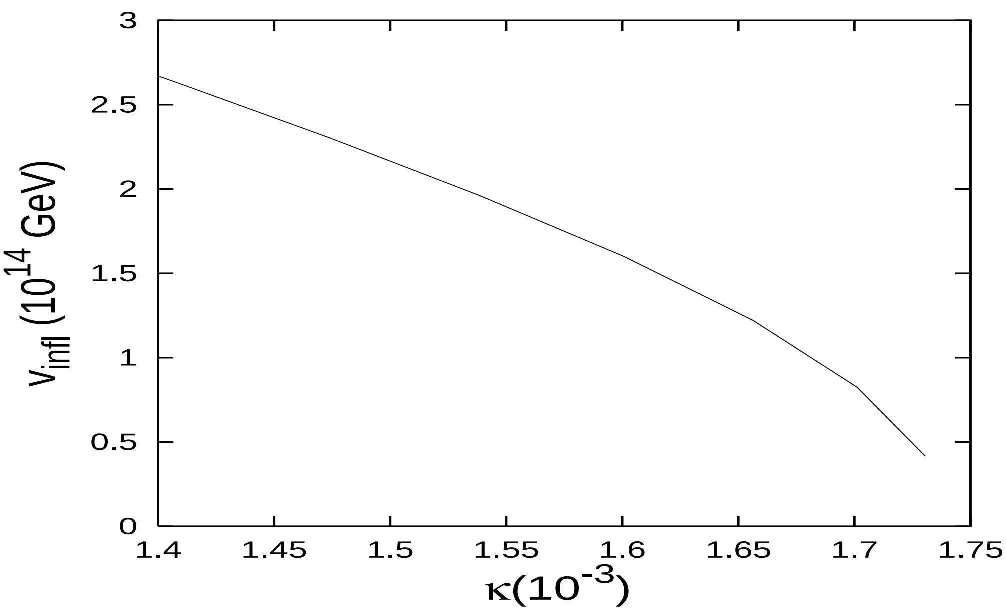

Figure 5: The inflationary scale $v_{\rm infl}$ as a function of the coupling constant κ.

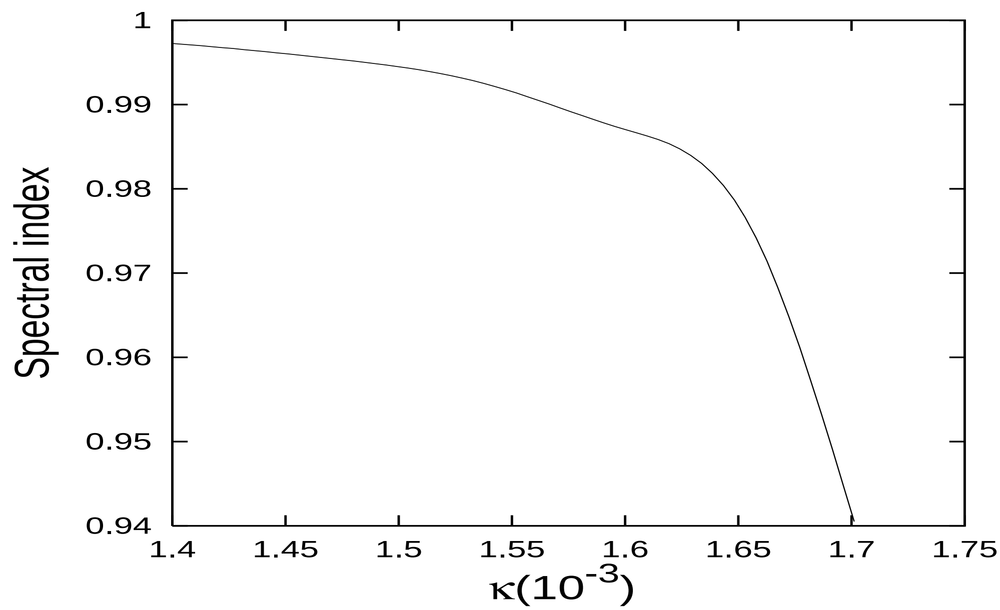

Figure 6: The spectral index n as a function of the coupling constant κ.

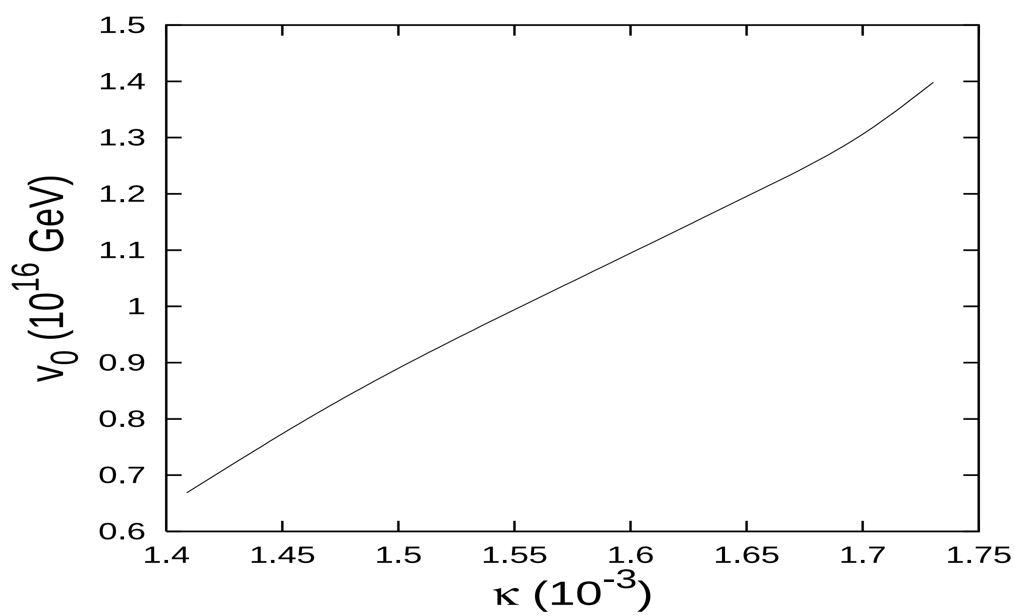

Figure 7: The common vev v_0 of $\bar{H}^c$, H^c at the SUSY minimum as a function of the coupling constant κ.